A PROPOSED TAXONOMY OF MATHEMATICAL VOCABULARY

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tax-on-o-my (tak-son'- -mi). n. 1. the science of classification; laws and principles covering the classifying of objects.
classify (klas'- -fi'). v.t. 1. to arrange or group in classes according to some system or principle.

Biologists make use of taxonomies to delineate the interrelationships between various plants and animals in the attempt to make their organization more meaningful and understandable. Mathematicians also use taxonomies, as when illustrating the interrelationships in number theory (see Figure 1). The arrangement of a taxonomy reflects interrelated characteristics of its elements and their relative complexity. The precision of language used in such taxonomies is intended to make communication more precise and, therefore, more effective.

Need for Vocabulary Taxonomy

Most teachers of mathematics would agree that a student's major difficulty when studying mathematics is the mastery of "vocabulary." Mathematics teachers say this as if it was a global difficulty, unaware of the existence of various levels of vocabulary type and complexity. A taxonomy could provide an outline for the various types of vocabulary, and would suggest their relative interrelationships. How would this help the teaching of mathematics?

Preview Materials. The teacher could preview materials to identify potentially troublesome words. Once teachers are aware that certain terms are relatively more difficulty to master because of the peculiarities of their function, such terms could be taught prior to the time when the student is asked to read mathematical material for meaning. Standard mathematics instruction would be rendered more effective because mathematical terminology would be taught with a focus on *function*, not as a global entity.

Diagnosis. Taxonomy levels could assist the teacher in informally diagnosing student difficulties. Many students may reveal, during instruction, a pattern of difficulty with a particular type of vocabulary.

Prescription. Taxonomic precision could be used to identify or develop materials that would be prescriptive of assessed weaknesses. Likewise, the teacher could direct pointed, taxonomically identified, terms to students with diagnosed weaknesses.

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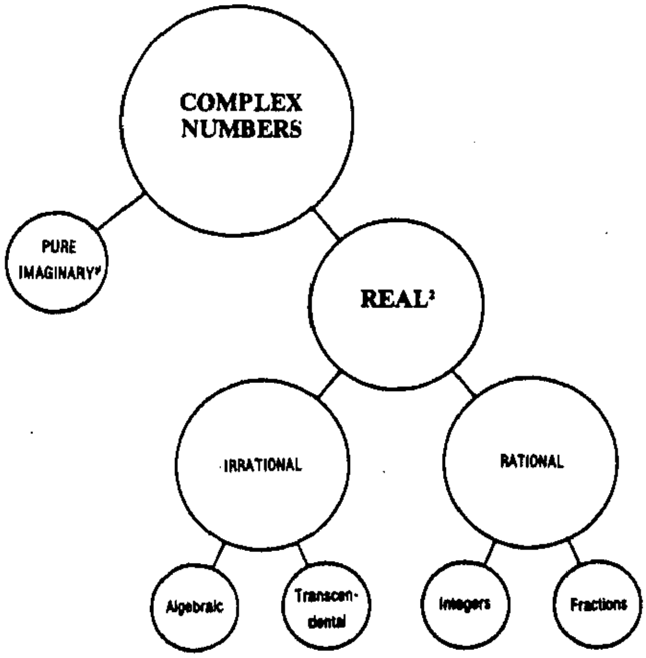
Scope and Sequence of Vocabulary Curriculum. Used as a curriculum guide, the taxonomy could outline and define the range of levels to be used when planning a full and continuous range of vocabulary teaching as a part of normal instruction.

Material Preview. Potential materials could be reviewed prior to purchase using the taxonomy as an evaluative tool. Those materials which provide effect development of higher levels of vocabulary could be separated from those less adequate.

Communication. The more precise definition provided by the taxonomy would certainly enhance more effective communication about the nature of vocabulary within the area of

Figure 1.

THE SYSTEM OF COMPLEX NUMBERS



[1]Numbers of the form a + bi, in which i is imaginary
[2]Numbers of the form a + bi, in which i is imagianry and b = 0
[3]Numbers of the form a + bi, in which i is imaginary and a = 0

Marks, Robert W., "The System of Complex Numbrs," *The New Mathematics Dictionary and Handbook,* Bantam Books: New York, 1964, p. 100.

mathematics. Teachers and researchers would be able to better communicate about the precise nature of student vocabulary difficulties, the need for instructional revision, the usefulness of particular instructional techniques, and, thus, greater strides could be made to render vocabulary instruction more effective.

Word Problem Vocabulary Difficulty. A taxonomy could also be helpful in further defining readability level as it relates to mathematical text materials and word problem solving. Beyond the problem-solving competencies involved in word problem solving, the more elusive factor of readability confounds the mathematics educator's efforts to level word problems by difficulty. The taxonomy has the potential of allowing the mathematics educator to give word problems (and indeed all mathematics materials) a "rigor level" based on vocabulary difficulty. This would offset the somewhat superficial readability formula valuation of difficult words being multisyllabic words. Vocabulary difficulty could be explored in terms of vocabulary function, a more complex valuation.

Relevant Research

Previous researchers have attempted to use the idea of vocabulary types. Packtman and Riley discussed four types of vocabulary involved in word problem solving: technical, symbols, everyday, and general. General use words are those involved in many situations that maintain a common mathematical meaning (example: *find* and *equal*). Technical terms are those whose meanings are specific to mathematics (examples: *integers* and *consecutive*). Everyday terms would be those non-mathematical words commonly used in oral and written language. Symbols included all numbers or graphic representations of mathematical concepts. The discussion within this article was limited to the solution of mathematical word problems and did not propose that any one type might be more difficult for students than the vocabulary of another type.

Dunlap and McKnight; considered a three-level translation of vocabulary which included *general, technical,* and *symbol* terminology. They asserted that "the translation process among vocabularies and the thinking process within each vocabulary are essential to the conceptualization of the message contained in the word problem." They view the solution process as (a) perceiving, (b) decoding, and then (c) translating into an almost "visual" concept all of the general vocabulary. This is then translated into technical mathematical context, and finally into a symbolic (computational) representation. A strength of their view involves the focus on meaning and visual conceptualization, but the visualization of terms which are commonly known in real life (for example, table, exercise) can cause mistranslation when encountered in a mathematical context (example: Use this *table* to solve the following *exercises.*). The terminology used in mathematics takes on differing functions, depending on context and vocabulary level. There is a need for specificity and definition when discussing effective use of mathematical terminology.

A Proposed Taxonomy

When examining the vocabulary within mathematics, five levels seem to recur. These levels appear to have interactive properties; that is a single term can be included in a number of categories, depending on context. This renders the levels somewhat overlapping, rather than exclusive. The various levels are discussed below, and are shown in abbreviated form in Figure 2.

Figure 2. A Taxonomy of Mathematical Vocabulary

I.	Standard	Common words normally encountered in oral and written language; i.e. this, that, over, above, etc.
II.	Transitional	Words which have both a mathematical referent as well as a common referent whose meanings are not the same; i.e. table, exercise, measurement
III.	Technical	Words specific to mathematics; includes *abbreviations* and *symbols;* i.e. addend, decimal, divisor, fraction, triangle, lb., oz, +, -, inverse
IV.	Changeable	Tecnical terms whose meanings change *within* the area of mathematics; i.e. square, prime, complementary, -, etc.
V.	Phrases	Multiple word terms with specialized meanings as a *unit,* the meaning of the unit is more than a compilation of the meanings of the parts; i.e. concrete number, square number, acute triangle, counting number, etc.

(1) *Standard* words are those the student would encounter in any type of written or oral language. Their meanings would tend not to differ widely in the area of mathematics from their normal, everyday definitions. These terms are the words encountered in dialogue, reading, or writing. They might include such words as: *is, word, there.*

The poor reader has difficulty operating on the standard level of mathematical terminology, and would not be expected to be able to deal effectively with higher levels of vocabulary. Students who experience difficulty with these words would not be able to get much information from text materials without some instructional adjustment. The teacher needs to devise a system by which materials can be orally presented to the poor reader. Two students, the poor reader and a more able reader, can be temporarily paired. The more able student can read important text passages aloud to the reading disabled student. Tape recordings of this oral presentation can be made and reused in other classes for the same purpose. Study guides, consisting of pretaught vocabulary and questions of sequenced difficulty which are keyed to the text location can be used to direct the disabled student's reading to help overcome text difficulty. The student's task becomes that of skimming for selected information, thus eliminating much of the disabling nature of the text material. Although time consuming, materials can be rewritten, drawing the readability level down to a level more in line with the student's ability. With reading disabled students, it is important that tests be given orally; otherwise the student's ability to demonstrate content knowledge will be masked by their reading deficiency.

(2) *Transitional* terms are encountered in the reader's daily experiences, but their meaning is decidedly different in the transition to a mathematical context. *Table,* for example, may evoke the image of a type of furniture instead of charted information that the mathematics teacher intended. Following directions like, "Use the table to solve the following exercises," may cause confusion for the student entering the mathematics classroom from physical education where *exercise* takes on a different meaning.

It is important for the teacher to preselect potentially troublesome transitional words and determine if they may be causing confusion for students. Otherwise, we may have a recurrance of the experience of the teacher who, directing students to "use their measurements to find the perimeter of the rectangle," had a pubescent reaction from one young student, "How can I use *my* measurements to find the perimeter of the rectangle?" Her preoccupation with her own body measurements interfered with her understanding of the term in a mathematical context.

(3) *Technical* terminology is readily acknowledged by mathematics teachers as the level of vocabulary most needing review to insure mastery. Technical terms are word encountered only in a mathematical context, such as *hexagon, divisor,* or *inverse.* Mastering these terms can be likened to learning a foreign language. Since these terms aren't used in everyday conversation, they are difficult to remember because they aren't practiced very often. Technical terms are learned only when maximally *used* by the *student.* The predominantly expository situation in many mathematics classrooms, however, is at odds with this principle of *student use.* The teacher generally introduces the terms, defines them, and uses them in an illustrative sentence. It is rare that students are given an opportunity to use them in their own verbalization. This is compounded by the structure of the textbook. Most terms are defined when they are first introduced. Rarely is a definition extended as the term is encountered on later pages; it is used as if the term were an old friend, or as if the definition were well understood and assimilated. To overcome this obstacle, mathematical terminology must be overlearned and used.

Activities like *20 Questions* can provide an opportunity for students to engage in discussion and practice. A student can think of a term (like *triangle*). The class is provided with a general category for the term (like *Geometry*), and is invited to ask any question about the unknown term that can be answered by yes/no. Questions like, "Does it have four sides?" or "Is it a closed figure?" will provide for meaning and terminology practice in a highly motivating way. The object is to identify the terms using fewer and fewer questions, teaching students the economy of questioning strategies.

Adaptations of popular television shows like *$25,000 Pyramid* can be used in mathematical vocabulary practice. Student pairs can present each other with clues to the identity of specific vocabulary terms. For example, the unknown word might be *triangle,* and clues might be: "Three sides, right," or similar phrases which will eventually evoke the term in question. Time limits are imposed to keep the practice lively.

Technical terminology also includes *symbols* and *abbreviations.* Abbreviations can be particularly difficult because they occasionally include none of the letters in the original word (as in *pound* = *lb.),* or may include additional letters not found in the original (as in *ounce* = *oz.*). The symbols of mathematics have their own unique problem. They must be instantly recognized to be understood. The student cannot apply phoneme (sound)-grapheme (letter) associations to "sound out" the pronunciation of a symbol in the same way a word can be attacked. Some symbols may also resemble other, more familiar, things to students. The use of *x* or • for multiplication may evoke the *letter x* and the *punctuation mark period* making the use of the symbols in their newer context more difficult to understand.

(4) *Changeable* terminology is similar to transitional vocabulary in that multiple meanings of words are involved, but the changes of meaning occur *within the area of mathematics* itself. For example, *square* refers to "a geometric figure" and "the result when a number is multiplied by itself" as in 4^2. Many symbols change their role depending on the mathematical context. The symbol "-" is *minus* in this context: *4-3 = 1;* but represents divide in this one: ¾. Later it is

used to indicate repeating decimals: $.\overline{333}$ and mean: x. It is also used within the symbol for divisions: 8 - 4 = 2. The context difference may only be a fraction of an inch placement on the page, a fine discrimination for students to confuse *b-d-p-q* and *on-no*. A middle school student was experiencing difficulty with changeable vocabulary when she was unable to deal with the multiplication of decimal numbers. The teacher was puzzled by the student's reaction, knowing she could multiply and had successfully dealt with addition and subtraction of decimals before. The problem came to light when the student was asked to read the problem aloud. The student refused, saying, "I can't read it aloud; I don't understand why there are two multiplication marks." Recall that in the context of multiplication of decimals, the notation is: 3 x .2 = □.

Teachers who are aware of the sequence of mathematics learning remember that students have encountered the same word with a different meaning prior to the instructional unit under consideration. They assess students' concepts of the word and take time to preteach the change before dealing with the new mathematical idea involved in this change of meaning.

(5) *Phrases* in mathematics have been identified as a problem.[5] They often appear to operate in an adjective-noun relationship; but, in fact, they project meaning only as a unit. Phrases can be detected by turning words around. A *red triangle* is a *triangle that is red;* the relationship is fairly direct and requires the insertion of few words to make the meaning clear when word order is reversed. Knowing that *acute* means "less than 90 degrees", it would be logical for an *acute triangle* to be a "triangle with less than 90 degrees." However, it actually means "a triangle *each of whose angles* is less than 90 degrees"—note the necessary addition of a great many concepts not subsumed within the terminology itself: Thus, a phrase is a case of the whole being more than just the sum of the parts. A *triangular number* does not mean "a number that is triangularly shaped." Nor does a *counting number* actually count, we *use it to* count. Mathematical dictionaries and textbook glossaries list these combinations as phrases. This unit-meaning relationship which occurs in phrases needs to be pointed out to students.

Conclusions and Implications

This interrelated taxonomy of mathematical vocabulary has implications for instruction, diagnosis, readability, problem solving, materials selection, the organization and development of instructional materials, curriculum development, and communication.

1. *Instruction*. As teachers become more familiar with the categories of the taxonomy, they would become more sensitive to the levels of terminology that might cause difficulties as students interact with mathematic materials. Teachers would know to preview materials to identify potentially troublesome terms which will need clarification or prior teaching.

2. *Diagnosis*. The taxonomy could be the basis of diagnosis, as a means of determining is specific students are experiencing difficulty at particular levels of the taxonomy. Materials could be identified or developed to provide remedial instruction for students who are unable to derive meaning from mathematical materials because of specific difficulties at a particular taxonomic level.

3. *Readability*. The taxonomy is a potential instrument for gauging the language difficulty (with relation to underlying mathematical *meanings)* of mathematical materials. This is a step beyond current readability formulae which tends to gauge difficulty on the basis of familiar words or number of syllables, not word function. Assuming the relative increase in difficulty of words within the taxonomy proves to be the case, this could be used to assign relative weights to vocabulary types to add a new dimension to readability formulae.

4. *Problem Solving*. This instrument would be helpful for categorizing word problems from lower to higher levels on the basis of the vocabulary content within the problem. This adds a further dimension to number and quality of operation, simple readability, abstractness, and other factors currently under examination to "rank" word problem difficulty.

5. *Materials Selection*. Teachers are provided an instrument which would be useful in selecting materials and tests appropriate for the learning levels of students. Materials heavy on phrases or changeable vocabulary would not be appropriate for student unable to deal with those language patterns. Materials which omit higher levels of vocabulary would, also, not be a prudent purchase, for they leave the student at the lowest levels of vocabulary understanding.

6. *Organization and Development of Instructional Materials*. The taxonomy offers suggestions to publishers as they develop new curriculular materials. It could provide a guide for sequencing levels of difficulty within mathematical texts. It could allow for key vocabulary to be drawn from text materials, and would suggest a more function-oriented means of dealing with key terms. The key terminology could be used to serve as a structured overview, which could serve as a sort of study guide to aid the students in discovering the interrelationships between concepts.

It also allows the mathematics teacher to convert real-life materials into instructional devices, rendering them useful motivators for classroom mathematics mateirals. Consider, for example, the mathematical terminology in a telephone or electric bill. A telephone book or rental lease all can, with the use of the taxonomy, yield realistic, motivating vocabulary practice.

7. *Curriculum Development*. Suggested vocabulary procedures could be critically viewed: "Do they provide for higher-level learning which involves multimeaning and phrase-level instruction?" The knowledgeable teacher would provide a full range of instruction at al levels of vocabulary. The taxonomy would guide curriculum specialists to a more comprehensive curriculum.

8. *Communication*. The taxonomy is a beginning to devising a more precise means of talking about the process(es) of mathematical vocabulary. With precise categories, discussions of vocabulary functions, instruction, deficits will be much more effective and task-focused.

Further work needs to be done to refine the taxonomy. More discreet categories, a more precise hierarchy would make such an endeavor more effective. Its use as a tool for sequencing word problems, in relation to their relative vocabulary difficulty, is strongly recommended. And, finally, level-specific instructional techniques should be developed.

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